Eric Rouse

Individual Assignments #58

Assignment: Section 1.6: 2, 4,6,14,16 (use the contrapositive), 18a

# Q2

If a is even and b is even then a+b is even.

P = a is even ⋀ b is even

Q = a + b is even

Assume P is true, a is even ⋀ b is even.

⟹ a = 2k; b = 2m

⟹ a + b ≡ 2k + 2m ≡ 2 (k+m) ⧠   
(because k+m is an integer, it follows the accepted definition of even numbers, a + b is even)

# Q4

If a is even then –a is even

P = a is even

Q = -a is even

Assume P is true, a is even.

⟹ a = 2k

⟹ -a ≡ -(2k) ≡ 2(-k) ⧠   
(because –k is an integer, it follows the accepted definition of even numbers, -a is even)

# Q6

If a and b are odd then ab is odd

P = a is odd ⋀ b is odd

Q = ab is odd

Assume P is true, a and b are odd.

⟹ a = 2k + 1; b = 2m + 1

⟹ ab ≡ (2k + 1)\*(2m+1) ≡ 4km + 2(k+m) + 1 ≡ 2\*(2km + k + m) + 1 ⧠   
(because (2km + k + m) is an integer, it follows the accepted definition of odd numbers, ab is odd)

# Q14

If x is rational and x != 0 then 1/x is rational

P = x is rational and x!= 0

Q = 1/x is rational

Assume P is true.

⟹x = a/b (rational numbers can be expressed as a ratio of integers)

⟹1/x ≡ 1/(a/b) ≡ b/a ⧠   
(b/a is a ratio of integers, hence 1/x is rational by the definition of rational numbers)

# Q16

If mn is even then m is even or n is even for all integers

P = mn is even

Q = m is even ⋁ n is even

Contrapositive: ¬Q → ¬P

¬Q ≡ ¬( m is even ⋁ n is even) ≡ ¬m is even ⋀ ¬n is even ≡ m is odd ⋀ n is odd; assumed to be true, so that m = 2a +1 and n = 2b + 1

¬P ≡ ¬(mn is even) ≡ mn is odd

⟹mn ≡ (2a+1)\*(2b+1) ≡ 4ab + 2(a+b) + 1 ≡ 2\*(2ab + a + b) + 1 ⧠  
(by the definition of odd numbers; mn is odd)

# Q18a

If 3n+2 is even then n is even for all integers.

P = 3n + 2 is even

Q = n is even

Contrapositive: ¬Q → ¬P

¬Q ≡ ¬(n is even) ≡ n is odd

¬P ≡ ¬(3n + 2 is even) ≡ 3n + 2 is odd

Substitute:

⟹ 3n + 2 ≡ 3(2a + 1) + 2 ≡ 6a + 4 + 1 ≡ 2(3a +2) + 1 ⧠  
(3n + 2 is odd by definition of odd numbers)